

## DEVELOPMENT OF PHYTOPLANKTON MODEL WITH APPLICATION TO SONGKHLA LAKE, THAILAND

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**ABSTRACT:** In this study, mathematical models are developed for simulating phytoplankton and nutrients dispersions in a water body. Two-dimensional vertically average mass balance equations of salinity, nitrogen and phosphorus in various forms and phytoplankton nitrogen are used as governing equations. The finite element method with Galerkin's weighted residual technique is used in model formulation. The developed models are applied to Songkhla Lake, which is one of the most important water resources in the south of Thailand, where algal bloom problem occurs in some months almost every year. Data on nutrient loadings from various sources in the watershed area are estimated based on the existing land use patterns, population density, livestock, aquaculture, and farming activities. Data on water depth and current velocities are obtained from a hydrodynamic model. The results from the model are illustrated to show distribution patterns of salinity, various forms of nitrogen and phosphorus, as well as phytoplankton nitrogen. The developed models can be used as a tool for assessing the effect of nutrient loading on algal bloom phenomena and are useful for water quality management planning of this lake and other water bodies.

**Keywords:** Phytoplankton, Water quality model, Finite element method, Nutrient dispersion model, Songkhla Lake.

### INTRODUCTION

The Songkhla Lake is one of the most important water resources in southern Thailand. It has very special ecosystems, comprising freshwater, brackish water and saline water ecosystems. Its watershed area covers about 8,754 km<sup>2</sup>, whereas the lake area is about 1,042 km<sup>2</sup>. In the past few decades, disposal of excessive pollutants from various activities around the lake has deteriorated water quality in the lake. One severe problem was the occurrence of algal bloom which adversely affected water quality and ecological balance in the lake (Laongsiriwong 2004; Ratanachai and Kanchanasuwan 2004).

The Songkhla Lake basin can be divided into 12 sub-basins, consisting of 8 sub-basins on the west coast and 4 sub-basins on the east coast. The whole basin is located between 6° 27' N. to 8° N. latitude, and 99° 44' E. to 100° 41' E. longitude. The Songkhla Lake can be divided into 3 main parts inter-connected by narrow canals, these are: 1) Upper lake: the largest part which has approximate area of 491 km<sup>2</sup> with average depth of 1.9 m. This part is freshwater ecosystem almost throughout the year except in some dry years when

salinity intrusion occurs. 2) Middle lake: the part next to the Upper lake. The surface area is about 336 km<sup>2</sup>. It is the shallowest part of the lake with average depth about 1.1 m. This part consists of freshwater and brackish water ecosystems. 3) Lower lake: the southern or the outer part of the Songkhla Lake which is connected to the Gulf of Thailand via a narrow channel. The surface area of this part is about 190 km<sup>2</sup>. The average depth is about 1.5 m. except along the strait for navigation where water depth is more than 7 m.).

For eutrophication restoration, many water quality models have been developed for providing necessary information for effective water quality management (Banjongraksa 2004; Sodsai 2004). Most of these models were developed to clarify the effects of various factors on eutrophication, especially the influence of nutrient loading on phytoplankton growth (Chao 2006; Gonanone 2004; Sarawuth et al. 2008; Umgiesser 2003). Some studies developed models for predicting the effects of salinity on plankton community (Dube et al. 2010; Ferreira 2005; Gasiünaitė 2005). Some researchers also considered the effects of salinity on biochemical processes such as nitrification rate (Zheng et al. 2004). The main nutrients of phytoplankton are nitrogen, phosphorus and carbon. Usually, there is enough carbon

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*Note:* Discussion on this paper is open until June 2013

source in natural water body whereas nitrogen and phosphorus are limited nutrients that affect phytoplankton growth. Fig. 1 shows the cycle of phytoplankton and limited nutrients in natural water body.

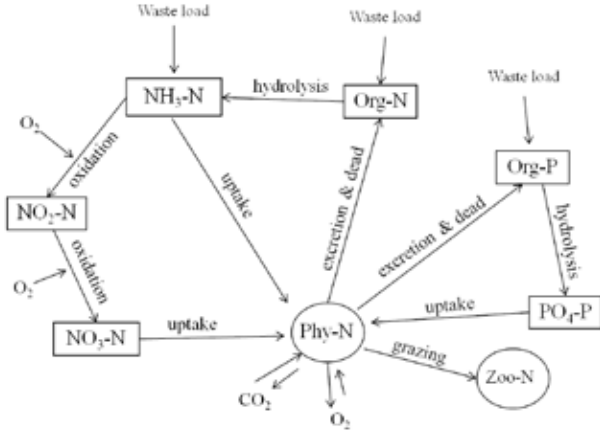


Fig. 1 Phytoplankton and nutrient cycle

## OBJECTIVES

The objectives of this study are to develop two dimensional water quality models to simulate temporal and spatial dispersions of phytoplankton and limited nutrients, and to apply the developed models to the Songkhla Lake. The parameters in the model include salinity, phytoplankton nitrogen, organic nitrogen, ammonia nitrogen, nitrite nitrogen, nitrate nitrogen, organic phosphorus and inorganic phosphorus.

## MODEL FORMULATION

### Formulation of a Dispersion Model

The basic governing equation of the dispersion model is the two-dimensional vertically averaged mass balance equation. This equation is used for all parameters included in the models. Only the source and the sink terms are different for different parameters. This mass balance equation is (Pritchard 1971):

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} - \frac{1}{h} \left\{ \frac{\partial}{\partial x} \left( h D_x \frac{\partial c}{\partial x} \right) + \frac{\partial}{\partial y} \left( h D_y \frac{\partial c}{\partial y} \right) \right\} - S_c = 0 \quad (1)$$

in which

- $c$  is the vertically averaged concentration of substance
- $u$  is flow velocity in the x-direction,
- $v$  is flow velocity in the y-direction

$h$  is total water depth

$D_x$  is dispersion coefficient in the x-direction

$D_y$  is dispersion coefficient in the y-direction

$S_c$  is the regeneration rate of substance per unit volume.

The finite element method with weighted residual technique is used to solve this partial differential equation. In this method the unknown variable  $c$  in Eq.(1) is replaced by a trial function  $\hat{c}$ , which is expressed in terms of its nodal values as follows:

$$\hat{c} = \sum_{i=1}^n N_i C_i = \mathbf{N}^T \mathbf{C} \quad (2)$$

in which

$N_i$  is interpolation function ( $i=1,2,..n$ )

$C_i$  is value of  $c$  at node  $i$  ( $i=1,2,..n$ )

$n$  is total number of nodes in the study area

$\mathbf{N}$  is column matrix of  $N_i$

$\mathbf{C}$  is column matrix of  $C_i$

The error or residual from this approximation is multiplied with a weighting function and the integral of the product over the whole study domain is set to zero (Connor and Brebbia 1976). This results in the following weighted residual equation:

$$\iint_{\Omega} w_c \left[ \frac{\partial \hat{c}}{\partial t} + u \left( \frac{\partial \hat{c}}{\partial x} \right) + v \left( \frac{\partial \hat{c}}{\partial y} \right) - S_c - \frac{1}{h} \left\{ \frac{\partial}{\partial x} \left( h D_x \frac{\partial \hat{c}}{\partial x} \right) + \frac{\partial}{\partial y} \left( h D_y \frac{\partial \hat{c}}{\partial y} \right) \right\} \right] dA = 0 \quad (3)$$

in which  $w_c$  is a weighting function.

By integrating by parts and applying Green's theorem, Eq.(3) can be transformed to

$$\iint_{\Omega} w_c \left[ \frac{\partial \hat{c}}{\partial t} + u \left( \frac{\partial \hat{c}}{\partial x} \right) + v \left( \frac{\partial \hat{c}}{\partial y} \right) - S_c \right] dA - \iint_{\Omega} \frac{w_c}{h} \left( D_x \frac{\partial h}{\partial x} \frac{\partial \hat{c}}{\partial x} + D_y \frac{\partial h}{\partial y} \frac{\partial \hat{c}}{\partial y} \right) dA + \iint_{\Omega} \left( D_x \frac{\partial w_c}{\partial x} \frac{\partial \hat{c}}{\partial x} + D_y \frac{\partial w_c}{\partial y} \frac{\partial \hat{c}}{\partial y} \right) dA - \int_S Q_c w_c dS = 0 \quad (4)$$

in which  $Q_c$  is the rate of substance dispersive flux per unit length through the domain boundary.

In Galerkin's technique, the interpolation function  $N_i$  ( $i=1, 2,..,n$ ) is used as the weighting function. If current velocities  $u$  and  $v$ , as well as water depth  $h$  at any location  $(x,y)$  are expressed in terms of their nodal values as  $\hat{c}$  in Eq.(2), then a set of first-order differential equations are obtained, which can be written in the matrix form as follow (Liengcharernsit, 1979):

$$\begin{aligned}
 & \iint_{\Omega} NN^T dA \frac{\partial C}{\partial t} + \left\{ \iint_{\Omega} N^T U \frac{\partial N^T}{\partial x} dA + \iint_{\Omega} N^T V \frac{\partial N^T}{\partial y} dA \right. \\
 & - \iint_{\Omega} \frac{N}{H} \left( D_x \frac{\partial N^T}{\partial x} H \frac{\partial N^T}{\partial x} + D_y \frac{\partial N^T}{\partial y} H \frac{\partial N^T}{\partial y} \right) dA \\
 & \left. + \iint_{\Omega} \left( D_x \frac{\partial N}{\partial x} \frac{\partial N^T}{\partial x} + D_y \frac{\partial N}{\partial y} \frac{\partial N^T}{\partial y} \right) dA \right\} C \\
 & - \iint_{\Omega} S_c N dA - \int_S Q_c N dS = 0
 \end{aligned} \quad (5)$$

in which

$U$  is column matrix of nodal flow velocities  $U_i$   
 $V$  is column matrix of nodal flow velocities  $V_i$   
 $H$  is column matrix of nodal water depths  $H_i$   
 $\bar{H}$  is average water depth

Eq.(5) can be written using matrix notation as:

$$M \frac{dC}{dt} + (M_{ux} + M_{vy} - M_{hx} - M_{hy} + M_{dx} + M_{dy})C - M_s - M_q = 0 \quad (6)$$

in which

$$M = \iint_{\Omega} NN^T dA \quad (7)$$

$$M_{ux} = \iint_{\Omega} NN^T U \frac{\partial N^T}{\partial x} dA \quad (8)$$

$$M_{vy} = \iint_{\Omega} NN^T V \frac{\partial N^T}{\partial y} dA \quad (9)$$

$$M_{hx} = \iint_{\Omega} \frac{K_x}{H} N \frac{\partial N^T}{\partial x} H \frac{\partial N^T}{\partial x} dA \quad (10)$$

$$M_{hy} = \iint_{\Omega} \frac{K_y}{H} N \frac{\partial N^T}{\partial y} H \frac{\partial N^T}{\partial y} dA \quad (11)$$

$$M_{dx} = \iint_{\Omega} D_x \frac{\partial N}{\partial x} \frac{\partial N^T}{\partial x} dA \quad (12)$$

$$M_{dy} = \iint_{\Omega} D_y \frac{\partial N}{\partial y} \frac{\partial N^T}{\partial y} dA \quad (13)$$

$$M_s = \iint_{\Omega} S_c N dA \quad (14)$$

$$M_q = \int_S Q_c N dS \quad (15)$$

In more compact form, Eq.(6) can be written as

$$M \frac{dC}{dt} + FC = M_s + M_q \quad (16)$$

in which

$$F = M_{ux} + M_{vy} - M_{hx} - M_{hy} + M_{dx} + M_{dy} \quad (17)$$

In the finite element method, the whole study domain is divided into a number of elements, associated with nodal points. The unknown variable in each element is expressed in terms of the values at nodal points of that element only. The integral over the whole study domain is obtained from summation of integrals over each element. Normally, the so-called natural coordinate system is used instead of the Cartesian coordinate system used in the original equation. Several types of elements can be selected together with the interpolation functions. With this natural coordinate system, the element matrices can be determined. These matrices are then assembled to form system matrices in Eqs.(7) - (15). In this study, a bilinear quadrilateral element is used.

#### Salinity Sub-model

Salinity is an important parameter which affects various chemical and biological processes in water. The growth of freshwater algae is inhibited when water salinity is higher. On the other hand, marine algae cannot survive in freshwater. Normally, the main source of salinity is from the sea and it is usually considered that salinity does not decay and is not consumed by aquatic flora and fauna, that is the source and sink terms of salinity can be set to zero, unless there is discharge of saline water into water body from some specific sources. So, the salinity model can be used to calibrate for the values of dispersion coefficients  $D_x$  and  $D_y$  which are important parameters in the dispersion model. The mass balance equation of salinity can be written as

$$\frac{\partial S}{\partial t} + u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} - \frac{1}{h} \left\{ \frac{\partial}{\partial x} \left( h D_x \frac{\partial S}{\partial x} \right) + \frac{\partial}{\partial y} \left( h D_y \frac{\partial S}{\partial y} \right) \right\} = 0 \quad (18)$$

in which  $S$  is salinity concentration

#### Phytoplankton Sub-model

There are several parameters used to measure the amount of phytoplankton such as chlorophyll-a, cell dry weight, and some major elements in phytoplankton cells. In this study, nitrogen within phytoplankton cells, which will be referred to as phytoplankton nitrogen, will be used to measure the phytoplankton biomass. The mass balance equation of phytoplankton nitrogen can be written as:

$$\frac{\partial P_n}{\partial t} + u \frac{\partial P_n}{\partial x} + v \frac{\partial P_n}{\partial y} - \frac{1}{h} \left\{ \frac{\partial}{\partial x} \left( h D_x \frac{\partial P_n}{\partial x} \right) + \frac{\partial}{\partial y} \left( h D_y \frac{\partial P_n}{\partial y} \right) \right\} - S_{pn} = 0 \quad (19)$$

where

$$S_{pn} = (G_{pn} - R_{pn})P_n - G_z Z - Q_{pn} \quad (20)$$

in which

- $P_n$  is phytoplankton nitrogen concentration
- $G_{pn}$  is growth rate of phytoplankton
- $R_{pn}$  is respiration rate of phytoplankton
- $G_z$  is phytoplankton nitrogen grazing rate per unit weight of zooplankton
- $Z$  is zooplankton concentration
- $Q_{pn}$  is increasing rate of phytoplankton nitrogen from other sources (if any).

#### Organic Nitrogen Sub-model

Organic nitrogen in various forms occurs from the discharge of organic pollutants and from dead and excretion of phytoplankton, zooplankton and other aquatic lives. The organic nitrogen will gradually decomposed to ammonia nitrogen. The mass balance equation of organic nitrogen can be written as

$$\frac{\partial O_n}{\partial t} + u \frac{\partial O_n}{\partial x} + v \frac{\partial O_n}{\partial y} - \frac{1}{h} \left\{ \frac{\partial}{\partial x} \left( h D_x \frac{\partial O_n}{\partial x} \right) + \frac{\partial}{\partial y} \left( h D_y \frac{\partial O_n}{\partial y} \right) \right\} - S_{on} = 0 \quad (21)$$

where

$$S_{on} = (U_{pn} - G_{pn} + R_{pn})P_n - R_{on}O_n + Q_{on} \quad (22)$$

in which

- $O_n$  is organic nitrogen concentration
- $U_{on}$  is nitrogen uptake rate per unit weight of phytoplankton nitrogen
- $R_{on}$  is mineralization/hydrolysis rate of organic nitrogen to ammonia
- $Q_{on}$  is increasing rate of organic nitrogen from other sources (if any).

#### Ammonia Nitrogen Sub-model

Ammonia nitrogen is utilized by phytoplankton as nutrient to form complex organic matter under photosynthesis. Another reaction which decreases the amount of ammonia nitrogen is its oxidation to nitrite

nitrogen under aerobic condition. The mass balance equation of ammonia nitrogen can be written as

$$\frac{\partial A_n}{\partial t} + u \frac{\partial A_n}{\partial x} + v \frac{\partial A_n}{\partial y} - \frac{1}{h} \left\{ \frac{\partial}{\partial x} \left( h D_x \frac{\partial A_n}{\partial x} \right) + \frac{\partial}{\partial y} \left( h D_y \frac{\partial A_n}{\partial y} \right) \right\} - S_{an} = 0 \quad (23)$$

where

$$S_{an} = R_{on}O_n - U_{an}P_n - R_{an}A_n + Q_{an} \quad (24)$$

in which

- $A_n$  is ammonia nitrogen concentration
- $U_{an}$  is rate of ammonia nitrogen uptake per unit weight of phytoplankton nitrogen
- $R_{an}$  is rate of ammonia nitrogen oxidation
- $Q_{an}$  is increasing rate of ammonia nitrogen from other sources (if any).

#### Nitrite Nitrogen Sub-model

Nitrite nitrogen occurs from oxidation of ammonia nitrogen under aerobic condition. The mass balance equation of nitrite nitrogen can be written as

$$\frac{\partial N_{ti}}{\partial t} + u \frac{\partial N_{ti}}{\partial x} + v \frac{\partial N_{ti}}{\partial y} - \frac{1}{h} \left\{ \frac{\partial}{\partial x} \left( h D_x \frac{\partial N_{ti}}{\partial x} \right) + \frac{\partial}{\partial y} \left( h D_y \frac{\partial N_{ti}}{\partial y} \right) \right\} - S_{ti} = 0 \quad (25)$$

where

$$S_{ti} = R_{an}A_n - R_{ti}N_{ti} + Q_{ti} \quad (26)$$

in which

- $N_{ti}$  is nitrite nitrogen concentration
- $R_{ti}$  is rate of nitrite nitrogen oxidation to nitrate nitrogen
- $Q_{ti}$  is increasing rate of nitrite nitrogen from other sources (if any).

#### Nitrate Nitrogen Sub-model

Nitrate nitrogen is utilized by phytoplankton as nutrient. It is the final product of nitrification process. Nitrate nitrogen occurs from oxidation of nitrite nitrogen. The mass balance equation of nitrate nitrogen can be written as

$$\frac{\partial N_{ta}}{\partial t} + u \frac{\partial N_{ta}}{\partial x} + v \frac{\partial N_{ta}}{\partial y} - \frac{1}{h} \left\{ \frac{\partial}{\partial x} \left( h D_x \frac{\partial N_{ta}}{\partial x} \right) + \frac{\partial}{\partial y} \left( h D_y \frac{\partial N_{ta}}{\partial y} \right) \right\} - S_{ta} = 0 \quad (27)$$

where

$$S_{ta} = R_{ti}N_{ti} - U_{ta}P_n + Q_{ta} \quad (28)$$

in which

$N_{ta}$  is nitrate nitrogen concentration

$U_{ta}$  is rate of nitrate nitrogen uptake per unit weight of phytoplankton nitrogen

$Q_{ta}$  is increasing rate of nitrate nitrogen from other sources (if any).

#### Organic Phosphorus Sub-model

Of the major plant nutrients, phosphorus is typically in limited supply in water, so it has high potential to limit phytoplankton growth. Phosphorus is chemically and biologically active under transformation between particulate and dissolved phases. The mass balance equation of organic phosphorus can be written as

$$\frac{\partial O_p}{\partial t} + u \frac{\partial O_p}{\partial x} + v \frac{\partial O_p}{\partial y} - \frac{1}{h} \left\{ \frac{\partial}{\partial x} \left( h D_x \frac{\partial O_p}{\partial x} \right) + \frac{\partial}{\partial y} \left( h D_y \frac{\partial O_p}{\partial y} \right) \right\} - S_{op} = 0 \quad (29)$$

where

$$S_{op} = (U_{ip} - G_{ip} + R_{op})P_n - M_{op}O_p + Q_{op} \quad (30)$$

in which

$O_p$  is organic phosphorus concentration

$U_{ip}$  is inorganic phosphorus uptake rate per unit weight of phytoplankton nitrogen

$G_{ip}$  is growth rate of phytoplankton multiplied with ratio of phosphorus to nitrogen in plankton cell

$R_{op}$  is respiration rate of phytoplankton multiplied with ratio of phosphorus to nitrogen in plankton cell

$M_{op}$  is mineralization/hydrolysis rate of organic phosphorus to inorganic phosphorus

$Q_{op}$  is increasing rate of organic phosphorus from other sources (if any).

#### Inorganic Phosphorus Sub-model

The mass balance equation of inorganic phosphorus can be written as

$$\frac{\partial I_p}{\partial t} + u \frac{\partial I_p}{\partial x} + v \frac{\partial I_p}{\partial y} - \frac{1}{h} \left\{ \frac{\partial}{\partial x} \left( h D_x \frac{\partial I_p}{\partial x} \right) + \frac{\partial}{\partial y} \left( h D_y \frac{\partial I_p}{\partial y} \right) \right\} - S_{ip} = 0 \quad (31)$$

where

$$S_{ip} = -U_{ip}P_n - R_{op}O_p + Q_{ip} \quad (32)$$

in which

$I_p$  is inorganic phosphorus concentration

$U_{ip}$  is rate of inorganic phosphorus uptake per unit weight of phytoplankton nitrogen

$Q_{ip}$  is increasing rate of inorganic phosphorus from other sources (if any).

#### WATER QUALITY ANALYSIS

In this study, water samples were collected at 41 stations which included 16 stations in canals drained from various sub-basins and 25 stations inside the lake (Fig. 2). Collection of water samples from canals drained

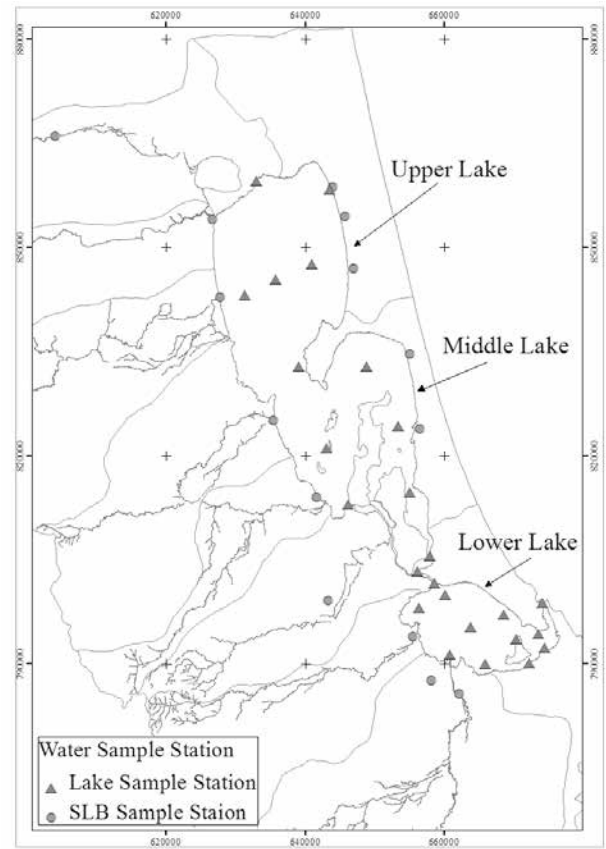


Fig. 2 Water quality sampling stations

from the basin was conducted monthly during April 2009 to May 2010. The water samples were analyzed for concentrations of chlorophyll-a, organic nitrogen, ammonia nitrogen, nitrite nitrogen, nitrate nitrogen, organic phosphorus, inorganic phosphorus, salinity, pH, dissolved oxygen. The analytical methods followed the Standard Methods for the Examination of Water and Wastewater (APHA *et al*, 1995). Besides, the contents of nitrogen and phosphorus in phytoplankton cells were also examined. The amount of nutrient concentrations in the canals draining into the Songkhla Lake are summarized in Table 1.

It was found that the concentrations of inorganic nutrients, i.e., inorganic nitrogen and inorganic phosphorus were higher than the concentrations of organic nutrients, i.e., organic nitrogen and organic phosphorus. In the Lower Lake, most nutrients were at higher concentrations than in the Upper lake and the Middle lake. This was due to higher discharge loading from communities located in the nearby sub-basins (La-ongsiriwong, 2004; Ratanachai and Kanchanasuwan 2004). Besides, concentrations of most nutrients were higher during the rainy season (from September to December), owing to higher nutrient loads from agricultural areas carried by surface runoff.

Table 1 Nutrient concentrations in the canals draining into the Songkhla Lake

Area		NH <sub>3</sub>	NO <sub>2</sub>	NO <sub>3</sub>	Org-N	PO <sub>4</sub>	Org-P
		(mg-N/l)	(mg-N/l)	(mg-N/l)	(mg-N/l)	(mg-P/l)	(mg-P/l)
Canals entering the Upper lake	mean	0.103	0.036	0.185	0.291	0.059	0.041
	S.D.	0.098	0.048	0.232	0.259	0.045	0.040
	min	0.006	0.002	0.013	0.004	0.004	0.001
	max	0.410	0.259	1.307	1.064	0.199	0.248
Canals entering the Middle lake	mean	0.110	0.020	0.084	0.326	0.042	0.034
	S.D.	0.156	0.024	0.076	0.229	0.032	0.027
	min	0.009	0.001	0.005	0.002	0.005	0.001
	max	0.770	0.136	0.459	1.045	0.152	0.109
Canals entering the Lower lake	mean	0.499	0.184	1.354	0.399	0.166	0.057
	S.D.	0.524	0.247	2.683	0.441	0.133	0.056
	min	0.010	0.001	0.102	0.008	0.005	0.001
	max	1.665	1.178	2.683	1.863	0.521	0.271
Total	mean	0.198	0.066	0.430	0.330	0.079	0.043
	S.D.	0.320	0.141	0.643	0.306	0.088	0.042
	Min	0.006	0.001	0.005	0.002	0.004	0.001
	Max	1.665	1.178	2.683	1.863	0.521	0.271

#### MODEL APPLICATION TO SONGKHLA LAKE

The developed phytoplankton & nutrient dispersion models are applied to the Songkhla Lake, one of the most important water resources in southern Thailand where algal bloom problem normally occurs in some months of each year. Details on model application can be explained as follow:

##### Element Configuration

The total area of 1,042 km<sup>2</sup> of the Songkhla Lake is divided into 138 quadrilateral elements with 220 nodes as shown in Fig. 3.

##### Model Input Data

The amounts of waste loads discharged into the Songkhla Lake are estimated based on existing land use patterns, population density, livestock, aquaculture, and farming activities. The input waste loads are distributed to the corresponding boundary elements of the study domain. The results obtained from the hydrodynamic model using the same element configuration are fed as input data of these water quality models. The models are run for a period of 1 month using time step of 300 seconds.



Fig. 3 Songkhla Lake divided into 138 elements

##### Model Calibration

The calibration of the dispersion models was carried out by comparing the results with measured field data and adjusting some parameters. Firstly, the salinity model was applied to the Songkhla Lake. Since the total dissolved salt can be considered as conservative substance, so the term representing decaying rate of salinity was set to zero. The salinity at the lake mouth boundary (about 30 ppt.) was specified as boundary value in the salinity model. From the model simulation it was found that the dispersion coefficient of 50 m<sup>2</sup>/sec

could provide satisfactory agreement between observed salinity and computed salinity (Fig. 4). The error was in the range of 0.001 – 3.9 ppt. with average percentage difference about 14.4%

Table 2 Important parameters used in model calculation.

	parameter	value
$G_{pn}$	growth rate of phytoplankton	0.06 day <sup>-1</sup>
$G_z$	phytoplankton nitrogen grazing rate per unit weight of zooplankton	5 m <sup>3</sup> /g
$R_{pn}$	respiration rate of phytoplankton	0.004 day <sup>-1</sup>
$R_{on}$	mineralization / hydrolysis rate of organic nitrogen to ammonia	0.15 day <sup>-1</sup>
$R_{an}$	rate of ammonia nitrogen oxidation to nitrite nitrogen	0.1 day <sup>-1</sup>
$R_{ni}$	rate of nitrite nitrogen oxidation to nitrate nitrogen	0.3 day <sup>-1</sup>
$R_{op}$	respiration rate of phytoplankton phosphorus	0.001 day <sup>-1</sup>
$PhyN\_Chl$	ratio of nitrogen in phytoplankton cell to chlorophyll- <i>a</i>	10
$P\_PhyN$	ratio of phosphorus to nitrogen in phytoplankton cell	0.1
$kmN$	Michaelis-Menton's constant for nitrogen limiting growth	0.03 µg-N
$kmP$	Michaelis-Menton's constant for phosphorus limiting growth	0.01 µg-P
$kSal$	Michaelis-Menton's constant for salinity limiting growth	0.07
$U_{An,max}$	ammonia nitrogen uptake rate per unit weight of phytoplankton nitrogen	0.06 hr <sup>-1</sup>
$U_{ta,max}$	nitrate nitrogen uptake rate per unit weight of phytoplankton nitrogen	0.03 hr <sup>-1</sup>

The value of dispersion coefficient obtained from the salinity model calibration was then used in the other models. The phytoplankton and nutrient dispersion models were then computed. The parameters in the models were calibrated within the normal ranges suggested by Gnananone (2004). The values of model parameters used in the simulation in this study are summarized in Table 2. It was found that the simulated results could provide satisfactory agreement with the measured field data.

## Results and Discussion

All the developed water quality models were run to simulate salinity, phytoplankton nitrogen and nutrient concentrations during November – December 2009. The results obtained from the models showed that there were some variations in concentrations of phytoplankton nitrogen as well as various forms of nitrogen and phosphorus between day and night in each day. This was due to the effects of sun light intensity on phytoplankton uptake and growth rates. However, these variations were rather small. The daily averaged spatial distributions of phytoplankton and various forms of nutrients are shown in Figs. 5 – 11. It was found that in the Upper lake and Middle lake, the amount of phytoplankton was higher than in the Lower lake (Fig. 5). It was due to shallower water depth and lower salinity value in the Upper and Middle lakes which were suitable for phytoplankton growth. On the other hand, concentrations of various nutrients in the Lower lake were higher (Figs. 5 - 11) due to higher waste loading as previously mentioned.

## CONCLUSION

In this study, mathematical models are developed to simulate dispersions of phytoplankton and their limited nutrients in a water body. The developed models are applied to the Songkhla Lake, one of the most important water resources in southern Thailand. Emphasis has been placed upon reliability of input data especially the amount of nutrient loads discharging into the lake. Therefore, field samplings and water quality analyses have been carried out in every month for the whole year (during April 2009 – May 2010). Model calibration is conducted to determine the suitable values of some model parameters. The results obtained from model show that water depth in the lake is the important factor which affects phytoplankton growth. Large amount of phytoplankton occurs in the shallow lake portion. However, the relationships between phytoplankton growth and nutrient concentrations are not so obvious in this study.

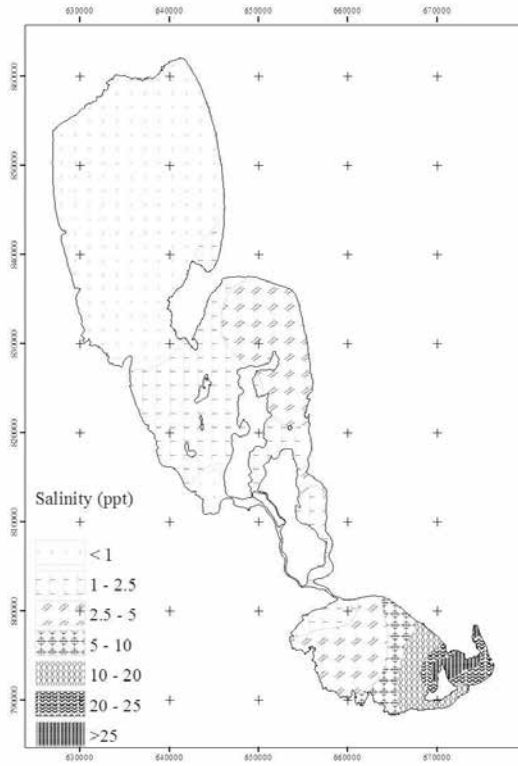


Fig. 4 Salinity distribution in the Songkhla Lake

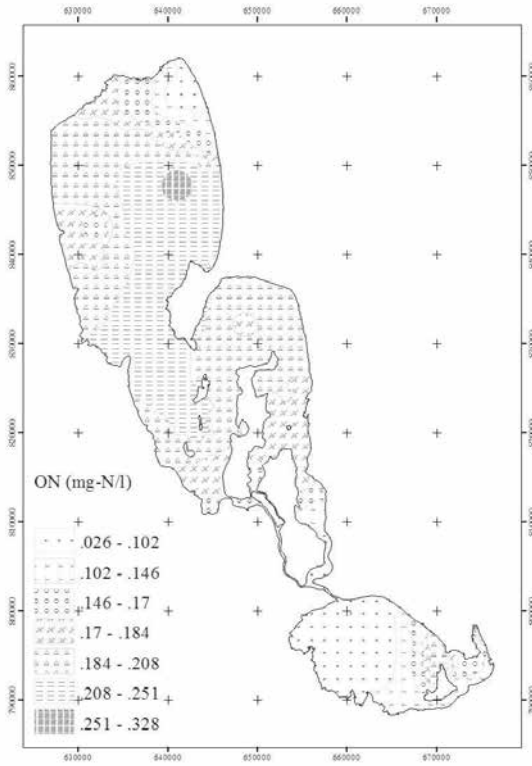


Fig. 6 Organic nitrogen distribution in the Songkhla Lake

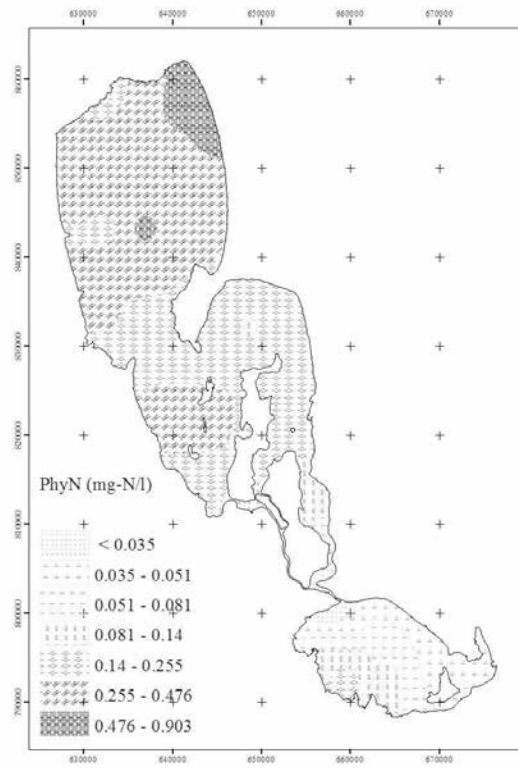


Fig. 5 Phytoplankton nitrogen distribution in the Songkhla Lake

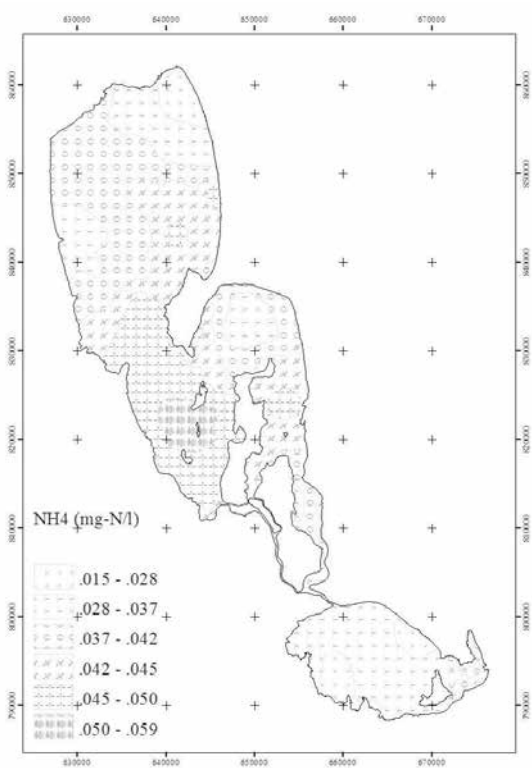


Fig. 7 Ammonia nitrogen distribution in the Songkhla Lake



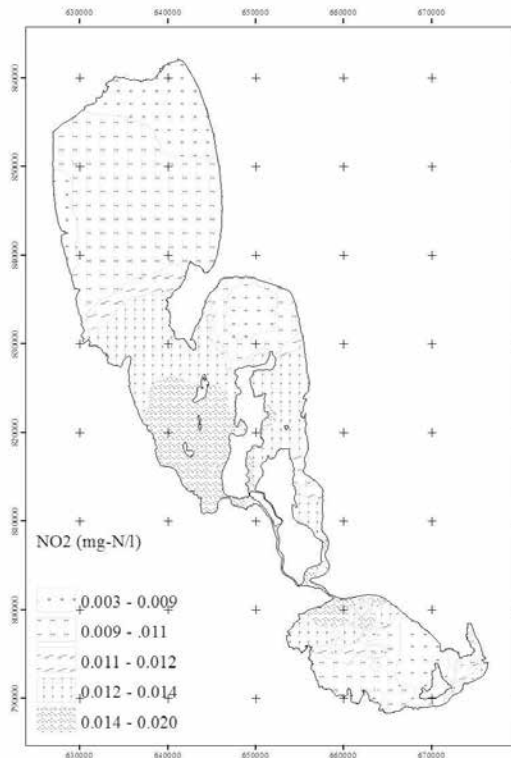


Fig. 8 Nitrite nitrogen distribution in the Songkhla Lake

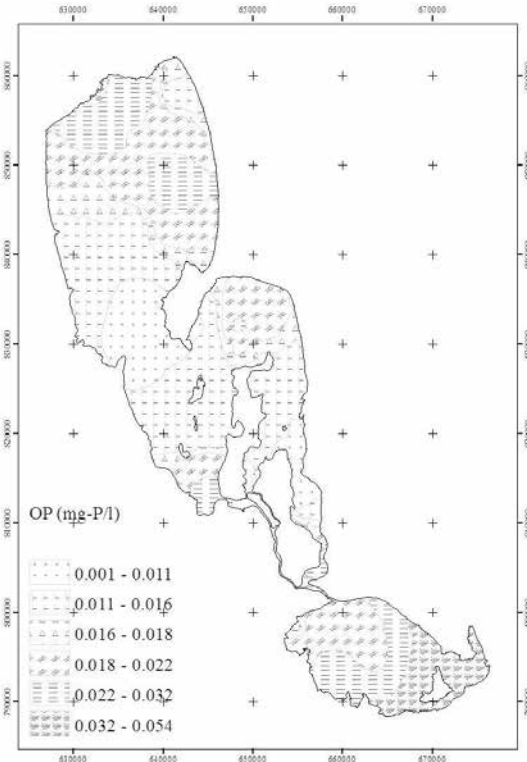


Fig. 10 Organic phosphorus distribution in the Songkhla Lake

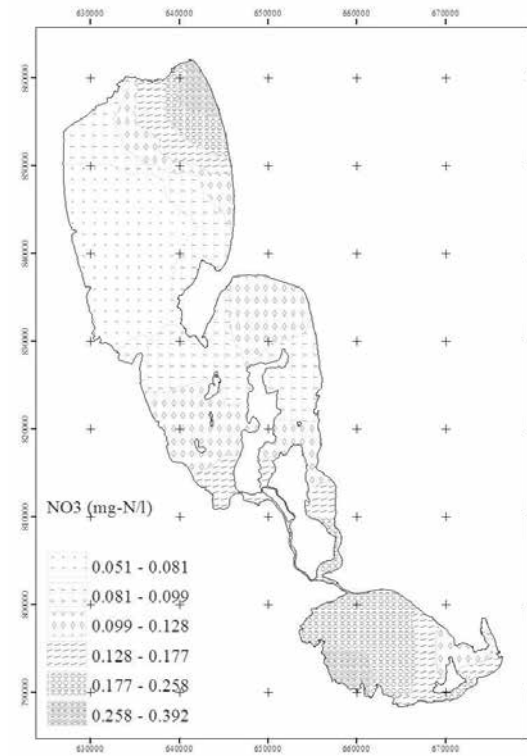


Fig. 9 Nitrate nitrogen distribution in the Songkhla Lake

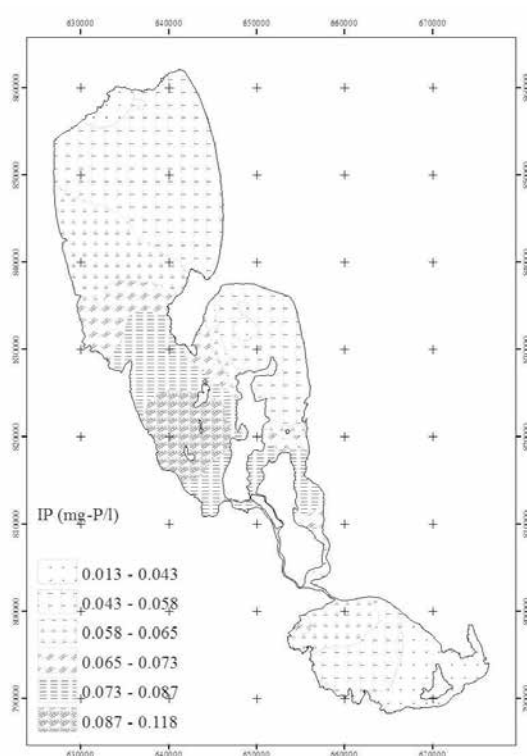


Fig. 11 Inorganic phosphorus distribution in the Songkhla Lake

## ACKNOWLEDGEMENTS

The authors are grateful to Prince of Songkhla University and the National Institute of Coastal Aquaculture for supporting facilities for field surveys and water quality analysis analyses. Also, they are grateful to the Graduate School of Kasetsart University and Thailand Toray Science Foundation for providing some financial support.

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